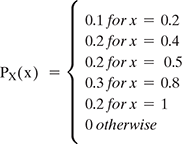
1. Given X be a discrete random variable with the following PMF



1. Find the range RX of the random variable X.

2. Find P(X ≤ 0.5)

3. Find P(0.25<X<0.75)

4. P(X = 0.2|X<0.6)

**Ans :**

1. The range of X from PMF. The range of X consists of possible values for X.

Rx = {0.2, 0.4, 0.5, 0.8, 1}

1. The event X<=0.5 can happen only if X is 0.2,0.4, or 0.5 Thus,

P(x<=0.5) = P(X belongs to {0.2, 0.4, 0.5}

=P(X=0.2)+P(X=0.4)+P(X=0.5)

=PX(0.2)+PX(0.4)+PX(0.5)

=0.1+0.2+0.2 = 0.5

1. The events between P(0.25<X<0.75)

= P(X belongs to {0.4, 0.5}

=0.2 + 0.2 = 0.4

1. Conditional probability P(X = 0.2|X<0.6)

=P((X=0.2) and X<0.6) / P(X<0.6)

=Px(0.2)/Px(0.2)+Px(0.4)+ Px(0.5)

=0.1/0.1+0.2+0.2

=0.2

2. Two equal and fair dice are rolled, and we observed two numbers X and Y.

1. Find RX, RY, and the PMFs of X and Y.

2. Find P(X = 2,Y = 6).

3. Find P(X>3|Y = 2).

4. If Z = X + Y. Find the range and PMF of Z.

5. Find P(X = 4|Z = 8).

**Ans:**

1. We have RX=RY={1,2,3,4,5,6}. Assuming the dice are fair, all values are equally likely so



PX(k)= 1/6 for k=1,2,3,4,5,

0 otherwise

Similarly for Y,

PY(k)= 1/6 for k=1,2,3,4,5,6

0 otherwise

1. Since X and Y are independent random variables, we can write

P(X=2,Y=6) = P(X=2)P(Y=6)

=1/6 \* 1/6=1/36.

1. Since X and Y are independent, knowing the value of Y does not impact the probabilities for X,

P(X>3|Y=2) =P(X>3)

=PX(4)+PX(5)+PX(6)

=1/6+1/6+1/6=1/2.

1. First, we have RZ={2,3,4,...,12}. Thus, we need to find PZ(k) for k=2,3,...,12. We have

PZ(2) =P(Z=2)=P(X=1,Y=1)

=P(X=1)P(Y=1) (since X and Y are independent)

=1/6 \* 1/6=1/36;

PZ(3) =P(Z=3)=P(X=1,Y=2)+P(X=2,Y=1)

=P(X=1)P(Y=2)+P(X=2)P(Y=1)

=1/6⋅1/6+1/6⋅1/6=1/18;

PZ(4) =P(Z=4)=P(X=1,Y=3)+P(X=2,Y=2)+P(X=3,Y=1)

=3⋅1/36=1/12.

We can continue similarly:

PZ(5) =4/36=1/9;

PZ(6) =5/36;

PZ(7) =6/36=1/6;

PZ(8) =5/36;

PZ(9) =4/36=1/9;

PZ(10) =3/36=1/12;

PZ(11) =2/36=1/18;

PZ(12) =1/36.

It is always a good idea to check our answers by verifying that ∑z∈RZPZ(z)=1. Here, we have

∑z∈RZPZ(z) =1/36+2/36+3/36+4/36+5/36+6/36+5/36+4/36+3/36+2/36+1/36

=1.

1. Note that here we cannot argue that X and Z are independent. Indeed, Z seems to completely depend on X, Z=X+Y. To find the conditional probability P(X=4|Z=8), we use the formula for conditional probability

P(X=4|Z=8) =P(X=4,Z=8) / P(Z=8)

=P(X=4,Y=4) / P(Z=8)

=P(X=4)P(Y=4) / P(Z=8) (since X and Y are independent)

=(1/6⋅1/6)/(5/36)

=1/5.

3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X's score is equal to the total number of correct answers, then find out the PMF of X. What is P(X>15)?

**Ans:**

Let's define the random variable Y as the number of your correct answers to the 10 questions you answer randomly. Then your total score will be X=Y+10. First, let's find the PMF of Y. For each question your success probability is 14. Hence, you perform 10 independent Bernoulli(14) trials and Y is the number of successes. Thus, we conclude Y∼Binomial(10,14), so

PY(y)={(10y)(14)y(34)10−y0for y=0,1,2,3,...,10otherwise

Now we need to find the PMF of X=Y+10. First note that RX={10,11,12,...,20}. We can write

PX(10) =P(X=10)=P(Y+10=10)

=P(Y=0)=(100)(14)0(34)10−0=(34)10;

PX(11) =P(X=11)=P(Y+10=11)

=P(Y=1)=(101)(14)1(34)10−1=10(14)(34)9.

So, you get the idea. In general for k∈RX={10,11,12,...,20},

PX(k) =P(X=k)=P(Y+10=k)

=P(Y=k−10)=(10k−10)(14)k−10(34)20−k.

To summarize,

PX(k)={(10k−10)(14)k−10(34)20−k0for k=10,11,12,...,20otherwise

In order to calculate P(X>15), we know we should consider y=6,7,8,9,10

PY(y)={(10y)(14)y(34)10−y0for y=6,7,8,9,10otherwise

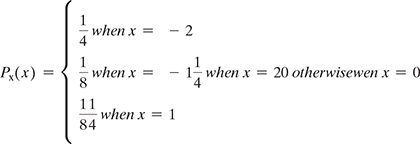
PX(k)={(10k−10)(14)k−10(34)20−k0for k=16,17,...,20otherwise

P(X>15)=PX(16)+PX(17)+PX(18)+PX(19)+PX(20)=(106)(14)6(34)4+(107)(14)7(34)3+(108)(14)8(34)2+(109)(14)9(34)1+(1010)(14)10(34)0.

4. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is P(10<Y≤15)?

5.Two independent random variables, X and Y,are given such that X~Poisson(α) and Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

6. There is a discrete random variable X with the pmf.



If we define a new random variable Y = (X + 1)2 then

1. Find the range of Y.

2. Find the pmf of Y.

**Ans:** Here, the random variable Y is a function of the random variable X. This means that we perform the random experiment and obtain X=x, and then the value of Y is determined as Y=(x+1)2. Since X is a random variable, Y is also a random variable.

To find RY, we note that RX={−2,−1,0,1,2}, and

RY ={y=(x+1)2|x∈RX}

={0,1,4,9}.

Now that we have found RY={0,1,4,9}, to find the PMF of Y we need to find PY(0),PY(1),PY(4), and PY(9):

PY(0) =P(Y=0)=P((X+1)2=0)

=P(X=−1)=18;

PY(1) =P(Y=1)=P((X+1)2=1)

=P((X=−2) or (X=0));

PX(−2)+PX(0)=14+18=38;

PY(4) =P(Y=4)=P((X+1)2=4)

=P(X=1)=14;

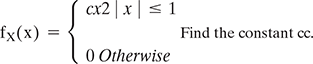
PY(9) =P(Y=9)=P((X+1)2=9)

=P(X=2)=14.

Again, it is always a good idea to check that ∑y∈RYPY(y)=1. We have

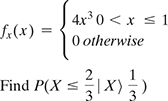
∑y∈RYPY(y)=18+38+14+14=1.

2.Assuming X is a continuous random variable with PDF

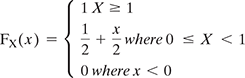


* + 1. Find EX and Var(X).
    2. Find *P*(*X* ≥ img).

1. If *X* is a continuous random variable with pdf

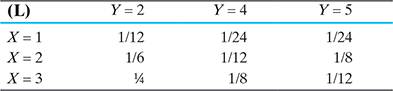


1. If *X*~Uniformimg and *Y* = sin(*X*), then find *fY*(*y*).
2. If X is a random variable with CDF



* + 1. What kind of random variable is *X*: discrete, continuous, or mixed?
    2. Find the PDF of *X*, f*X*(*x*).
    3. Find E(eX).
    4. Find P(*X* = 0|X≤0.5).

1. There are two random variables *X* and *Y* with joint PMF given in Table below
   * 1. Find *P*(*X*≤2, *Y*≤4).
     2. Find the marginal PMFs of *X* and *Y*.
     3. Find *P*(*Y* = 2|*X* = 1).
     4. Are *X* and *Y* independent?



6.A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.

7.If A and B are two jointly continuous random variables with joint PDF

images

a. Find fX(a) and fY(b).

b. Are A and B independent of each other?

c. Find the conditional PDF of A given B = b, fA|B(a|b).

d. Find E[A|B = b], for 0 ≤ y ≤ 1.

e. Find Var(A|B = b), for 0 ≤ y ≤ 1.